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Effect of different parameters on the behavior of strip footing resting on weak soil improved by granular piles

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Abstract

This paper presents a method to analyze strip footing resting on granular layer over weak soil improved by end bearing or floating granular piles. The granular layer beneath the strip footing idealized as incompressible shear layer. The weak soil idealized as soft Winkler springs and the granular piles idealized as stiff Winkler springs. These springs connected at their ends by a thin membrane under uniform tension to overcome the drawbacks of Winkler model related to the shear effect or the continuity of the granular piles-weak soil composite. Hyperbolic relationships used to represent the nonlinear behavior of weak soil and granular piles. Granular piles of different diameters, lengths, stiffness and arrangements can be modeled. The numerical solution obtained by the finite element method. The validity of the present analysis verified through comparisons with the field measurements, other existing analysis method and PLAXIS finite element program. A parametric study for a strip footing subjected to seven concentrated column loads carried out to study the effect of various parameters on the behavior of the soil-strip footing system. These parameters include number of granular piles, modular ratio, stiffness of granular layer, diameter of granular piles, length of granular piles and arrangements of granular piles. It is found that stiffness of granular layer has little effect compared to a significant effect of other parameters on the vertical and differential displacements and the induced bending moment of the strip footing.

Keywords: Finite element analysis, Strip footing, Granular layer, Weak soils, Floating granular piles

Background

Use of granular piles, GP, in weak soils (e.g., soft clay and loose sand) is now a well known ground improvement technique. In case of loose granular soil, the provision of granular pile enhances the bearing capacity of foundation and reduces its total and differential settlements. However, in case of soft cohesive soil, it has an additional advantage of providing a drainage path, which increases the rate of consolidation. Granular piles may be fully penetrated and resting on strong soil layer (i.e., end bearing granular piles, EBGP) or partially penetrated (i.e., floating granular piles, FGP). The floating granular piles are considered an economic alternative system to fully penetrated granular piles in case of deep weak soil layer or in case of lightly loaded structures. A granular fill layer of sand or sand-gravel mixture is usually placed over the top of granular piles reinforced weak soils [1].

Several literature pertaining to the behavior of footings resting on fully penetrated granular piles are found (e.g., [2–12]. But, a little number of literature concerning the behavior of footings resting on floating granular piles are found (e.g., [13–22]. For space limitations, only review the technical literature pertaining to the analysis of strip footing resting on weak soil improved by granular piles is presented in this section.

Deb et al. [4] proposed a mechanical model to predict the behavior of a geosynthetic reinforced granular fill over soft soil improved with end bearing granular piles. The granular layer, surrounding soil, and stone columns were idealized by Pasternak shear layer, Kelvin-Voight model, and stiffer Winkler spring, respectively. The plane strain condition was considered in the analysis and the finite difference scheme is used to solve the governing differential equations. Nonlinear behaviors of soft soil and the granular fill were considered. For a uniformly loaded strip footing, the presence of granular layer helps to transfer stress from soil to granular piles and reduce maximum and differential settlements [3, 4].

Maheshwari and Khatri [7, 8] proposed a nonlinear mechanical model for analysis of strip footing resting on granular layer over end bearing stone column reinforced earth beds. The granular layer, weak soil and stone columns were idealized by Pasternak shear layer, Kelvin-Voight model, and stiffer Winkler spring respectively. The flexural rigidity of strip footing and the nonlinearity of granular layer, stone column and soft soil were taken into consideration. The effect of different parameters on the behavior of soil-strip footing system was investigated. Maheshwari and Khatri [9] proposed a generalized model for analysis of strip footing on geosynthetic-reinforced granular fill over stone columns improved soft soil system. The granular layer, Geosynthetic layer, weak soil and stone columns were idealized by Pasternak shear layer, elastic membrane, Kelvin-Voight model, and stiffer Winkler spring respectively. The nonlinearity of granular layer, stone column and soft soil were taken into consideration.

Strip footings have finite flexural rigidity are usually analyzed as beams on elastic foundation. Many studies for the analysis of beams on elastic foundation were presented in the literature (e.g., [23–25]. In these studies, the two-parameter model or three-parameter model used to idealize the soil.

In all the studies pertaining to the analysis of strip footing resting on weak soil improved by granular piles, the weak soil and the granular piles were idealized as a series of independent vertical soft and stiff Winkler springs and neglect the shear interaction between springs or the continuity of granular piles-weak soil composite. In addition, these studies do not incorporate the effect of granular piles length (i.e., floating granular piles), granular piles arrangement and granular piles of different diameters on the strip footing behavior.

In this paper, a method is developed to analyze the strip footing resting on granular layer over weak soil improved by end bearing or floating granular piles. The nonlinear behavior of weak soil and granular piles are taken into consideration. Comparisons between the results of the present analysis with the field measurements, results of other existing analysis method and results of PLAXIS program are presented for the purpose of validation. The effect of different parameters on the behavior of soil-strip footing system is also investigated.

The problem under consideration

Figure 1 shows the definition sketch of a strip footing resting on a granular layer over top of granular piles improved weak soil. The strip footing is of width B and length L and subjected to a number of concentrated loads (i.e., $Q_1, Q_2, \ldots, Q_n$). The thickness of granular layer is H_{gl} and its shear modulus is G_{gl}. Diameter and spacing of granular piles are D_{gp} and S, respectively.

Figure 2 shows the proposed model for the soil-strip footing system under consideration. The strip footing is modeled as a finite beam of flexural rigidity, EI. The granular layer is idealized as Pasternak shear layer [26]. The weak soil and the granular piles are idealized as soft and stiff Winkler springs respectively. These springs are connected at their ends by a thin membrane under uniform tension to take into account the shear interaction between the Winkler springs [27, 28]. The length of the granular piles is assumed equal to the thickness of natural weak soil stratum (i.e., case of end bearing granular piles) or less than the thickness of weak soil stratum (i.e., case of floating granular piles). While installing the granular piles in weak soils, the original stiffness of ground will increase [15, 29]. However, this effect is not considered in the present analysis.

Method of analysis

In the literature, a number of two-parameter models are presented to overcome the weakness of Winkler model (i.e., the assumption that there is no interaction between adjacent springs) in modeling the behavior of elastic foundation. In these models, the first parameter represents the stiffness of vertical springs, as in the Winkler model, and the second parameter was introduced to account for the coupling effect between vertical springs. These two-parameter models were presented and discussed by a number of researchers [23, 27, 28].

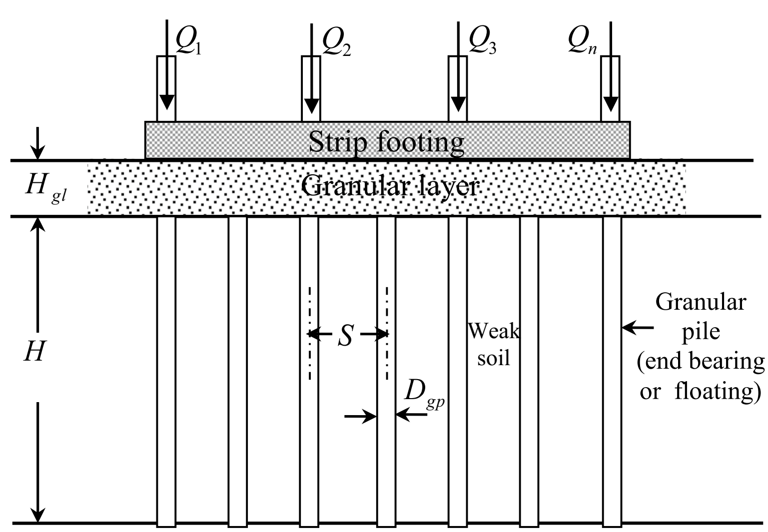

Fig. 1 The problem to be analyzed

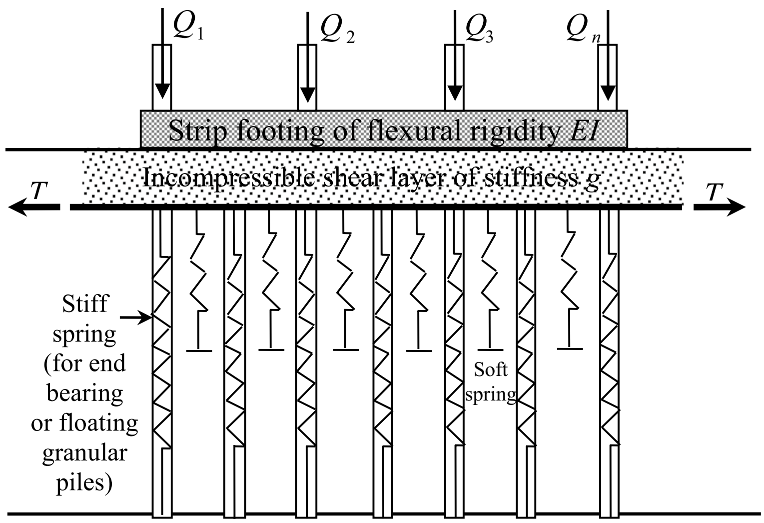

Fig. 2 The problem modeling

The problem of a dense coarse grained soil layer laying on a compressible soil can be idealized as an incompressible shear layer (i.e., Pasternak shear layer) of stiffness g over a weak soil reinforced by granular piles idealized as soft and stiff Winkler springs of modulus of subgrade reaction coefficients k_s and k_{gp} respectively. The soft and stiff Winkler springs are connected at their ends by a thin membrane under uniform tension force per unit length, T, to overcome the drawbacks of Winkler model related to the shear effects or the continuity of the soil mass. The governing equation of such a mechanical subgrade model is as follows [27, 30].

$$p = kw - (T + g)\frac{d^2w}{dx^2} \tag{1}$$

where p is the subgrade reaction, k is the modulus of subgrade reaction (i.e., $k = k_s$ over weak soil and $k = k_{gp}$ over granular piles) and w is the vertical displacement.

The differential equation of a beam is obtained by considering the bending of an elemental segment. The differential equation of the beam with uniform cross section in the absence of any external uniformly distributed load can be written as follows.

$$EI\frac{d^4w}{dx^4} + p = q \tag{2}$$

Combining Eqs. (1) and (2), the following differential equation of the soil-strip footing system is obtained.

$$EI\frac{d^4w}{dx^4} + kw - (T + g)\frac{d^2w}{dx^2} = q \tag{3}$$

where E is the modulus of elasticity of strip footing, I is the moment of inertia of strip footing and q is the applied transverse load on strip footing. The nonlinear behavior of weak soil and granular piles are expressed by hyperbolic stress–strain relationships as suggested by Maheshwari and Khatri [8].

$$k_s = k_{so}\left(1 - \frac{R_{fs}\sigma_s}{q_{su}}\right) \tag{4}$$

$$k_{gp} = k_{gpo}\left(1 - \frac{R_{fgp}\sigma_{gp}}{q_{gpu}}\right) \tag{5}$$

where k_{so} and k_{gpo} are the initial values of modulus of subgrade reactions of weak soil and granular pile, σ_s and σ_{gp} are the stresses on weak soil and granular pile, q_{su} and q_{gpu} are the ultimate bearing capacities of weak soil and granular pile, R_{fs} and R_{fgp} are the hyperbolic curve fitting constants for weak soil and granular pile respectively. In the present analysis the length of the granular pile is generally greater than 6 times its diameter (i.e., long granular piles) and therefore, the value of q_{gpu} is calculated based on the bulging deformation of the granular pile [31–33].

The initial modulus of subgrade reaction of weak soil can be calculated by one of the methods presented in the literature [34]. Here, the initial modulus of subgrade reaction is calculated from the following equation [28].

$$k_{so} = \frac{E_s(1 - \nu_s)}{H(1 - \nu_s - 2\nu_s^2)} \tag{6}$$

where E_s and ν_s are the modulus of elasticity and Poisson's ratio of weak soil layer and H is the depth of influence.

The depth of influence is the smaller depth of either the depth of weak soil below foundation level to the rigid base or the depth below foundation level at which the settlement caused by foundation pressure equal to zero [30]. The value of H is dependent on beam dimensions, relative rigidity of the beam with the soil and load pattern acting on the beam and can be taken in the range of 2–4 times beam width [28, 30].

For simplicity, the value of the second parameter, T, is calculated based on the assumption that the granular piles-soil composite behaves like a uniform soil mass with composite modulus of elasticity and Poisson's ratio, E_{comp} and ν_{comp}, as follows [28]. Such simplification used by Priebe [10] to calculate the shear values of the improved ground.

$$E_{comp} = A_r E_{gp} + (1 - A_r)E_s \tag{7}$$

$$\nu_{comp} = A_r \nu_{gp} + (1 - A_r)\nu_s \tag{8}$$

$$A_r = \frac{BL}{N_{gp}A_{gp}} \tag{9}$$

$$T = \frac{E_{comp}H}{3(1 + \nu_{comp})} \tag{10}$$

where E_{gp} and ν_{gp} are the modulus of elasticity and Poisson's ratio of granular piles, A_r is the area replacement ratio, N_{gp} is the number of granular piles, A_{gp} is the cross sectional area of granular pile, and B and L are the width and the length of the strip footing.

The stiffness of incompressible shear layer (i.e., granular layer) can be calculated from the following equation [27, 28].

$$g = \frac{G_{gl}H_{gl}}{2} = \frac{H_{gl}}{2}\left(\frac{E_{gl}}{2(1+\nu_{gl})}\right) \quad (11)$$

where H_{gl} and G_{gl} are the thickness and shear modulus of granular layer. E_{gl} and ν_{gl} are the modulus of elasticity and Poisson's ratio, ν_{gl}, of the granular layer.

For end bearing granular piles, the coefficient k_{gpo} can be calculated as the calculation of the coefficient k_{so} as follows:

$$k_{gpo} = \frac{E_{gp}(1-\nu_{gp})}{H(1-\nu_{gp}-2\nu_{gp}^2)} \quad (12)$$

where the parameters of Eq. (12) as defind above.

Partially improved ground with granular piles and the underlying compressible weak soil create a double-layered compressible foundation. So far, no reasonable solution is available to estimate the modulus of subgrade reaction of such a double-layered foundation. In the present study, the initial modulus of subgrade reaction of floating granular pile, k_{fgpo}, is calculated from the following equation:

$$k_{fgpo} = \frac{E_{eq}(1-\nu_{eq})}{H(1-\nu_{eq}-2\nu_{eq}^2)} \quad (13)$$

where E_{eq} and ν_{eq} are the equivalent modulus of elasticity and equivalent Poisson's for a double-layered compressible foundation. The equivalent homogeneous, isotropic value of E_{eq} and ν_{eq} are determined using the weighted average approach.

Finite element formulation

The strip footing is divided into a number of elements (i.e., 4 degrees of freedom beam element) taking into account the locations of granular piles to be at the elements nodes. Using the standard procedures in the finite element method for the assemblage of elements, the global stiffness matrix is constructed as a half banded matrix. In matrix formulation, the differential equation, Eq. (3), can be expressed as follows:

$$[K]\{W\} = \{F\} \quad (14)$$

$$[K] = \sum_{i=1}^{N_e}\left[(K_b) + (K_s) + (K_T) + \left(K_g\right)\right] \quad (15)$$

where $[K]$ is the global coefficient matrix, $\{W\}$ is the global nodal displacements; and $\{F\}$ is the global nodal external load vector of the system, (K_b) is the stiifness matrix of the flexure beam element, (K_s) is the first foundation stiffness matrix to account the effect of k_s, (K_T) is the second foundation stiffness matrix to account the effect of T and $\left(K_g\right)$ is the stiffness matrix of incompressible shear layer to accound the effect of g.

The stiffness matrix of the beam element, the subgrade parameters (k_s, T) and incompressible shear layer parameter, g, were presented in the literatur (e.g., [27, 35]. The spring stiffness of the granular piles added to the corresponding places on the diagonal of the global stiffness matrix. Applying the proper boundary conditions, we get the solution of the deformations (i.e., vertical displacements and rotations) in the strip footing. These deformations are used to determine the internal forces in the strip footing (i.e., shear forces and bending moments), contact pressure and the nodes reactions.

At the edge of the beam special boundary condition is required to replace the subgrade effects beyond the edge of the beam. Colasanti and Horvath [30] suggested an additional independent axial spring under the edge of the beam (i.e., at the level of weak soil springs). The stiffness of these additional boundary condition springs can be calculated from the following equation [30].

$$k_{bc} = \sqrt{k_s T} \tag{16}$$

Results and discussion

A computer program is developed based on the finite element method to analyze the soil-strip footing system under consideration using the above methodology. The developed program is able to calculate vertical displacements, rotations, shear forces, bending moments, contact pressure, nodes reactions. The analysis procedure is general enough to take into account different lengths, diameters, and stiffness of granular piles, any arrangements of granular piles and any types of external loads acting on the strip footing (i.e., concentrated loads, uniformly and non-uniformly distributed loads and moment loads).

Validation

For the purpose of validation, comparison between the predicted values by the present method with the field measurements, the results of other existing analysis method and the results of PLAXIS program are made.

Comparison with field measurements

Watts et al. [36] carried out a full-scale instrumented load tests to study the performance of end bearing stone columns supporting a strip footing in a variable fill and the performance of a similar strip footing on untreated ground. Watts et al. [36] presented soil profile, results of various in situ and laboratory tests and instrumentation. The dimensions of treated and untreated strip footings were 9 m length, 0.75 m width and 0.25 m thickness and subjected to three different uniformly distributed loads. Here, only comparison with the uniformly distributed load of 123 kPa is considered. The number, diameter and spacing of stone columns were 9, 0.6 and 1.8 m, respectively. Thickness of the treated soil below the foundation level varies from 3.15 m at left edge to 4.35 m at right edge with an average thickness of 3.75 m. Lengths of stone columns varied with the thickness of the treated soil. The modulus of elasticity of untreated soil and stone columns were 5 and 30 MPa, respectively [36]. Poisson's ratio of the soil and the granular piles are taken equal to 0.35 [37]. The modulus of subgrade reaction of the soil and the second parameter, T, are calculated from Eqs. (6) and (10) respectively. The modulus of subgrade reaction of stone column is taken 6 times the modulus of subgrade reaction of the soil [3], where 6 is the ratio between E_{gp} and E_s. Linear analysis is considered. Figure 3 shows comparisons between measured and predicted vertical displacements for treated and untreated strip footings.

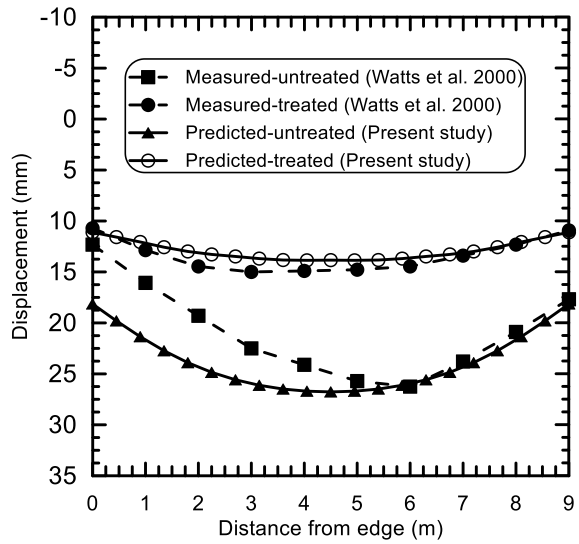

Fig. 3 Comparison between measured and predicted settlements for untreated and treated strip footings

For untreated strip footing, the best match between measured and predicted vertical displacements is obtained at the value of the depth of influence equal to 1.65 times width of the strip footing as shown in Fig. 3. The difference between the present results and the measured values at the left part of the strip footing is due to the fact that in the present analysis a constant soil layer is considered, whereas in the field the soil thickness is varied along the beam length. However, for treated strip footing, the predicted values by the present analysis are compared well with the measured vertical displacements at the edges and slightly smaller than the measured values at the middle part of the strip footing as shown in Fig. 3. One of the drawbacks of Winkler model is that a strip footing subjected to a uniformly distributed load will undergo rigid body displacements without any shear forces or bending moments in the strip footing. The results obtained by the present analysis for case of untreated strip footing reveals that the importance of using two-parameter model to represent the soil instead of using one-parameter model (i.e., Winkler model).

Comparison with other existing analysis method

Maheshwari and Khatri [8] developed a method for the analysis of strip footing resting on granular layer over weak soil reinforced by granular piles. The present method is validated by comparing its results with the results from Maheshwari and Khatri [8]. The strip footing is of flexural rigidity $EI = 150090.7$ kN m^2 and subjected to five equal concentrated loads. The granular piles diameters are 0.3 m and its spacing is 0.9 m. The thickness of the granular layer and its shear modulus are 0.5 m and 380 kN/m^2, respectively. The coefficients of subgrade reaction of weak soil and granular piles are 10,000 kN/m^3 and 100,000 kN/m^3, respectively [8]. Maheshwari and Khatri [8] ignored the second parameter, T, and therefore T is taken equal zero in the present analysis. Linear and nonlinear analysis is considered.

Figure 4 shows comparison between vertical displacements obtained by the present analysis with those obtained by Maheshwari and Khatri [8] for linear and nonlinear cases. Generally, good comparisons are obtained for linear and nonlinear cases as shown in Fig. 4. The little difference between the present results and the results presented by Maheshwari and Khatri [8] is due to the fact that in the present study the stiffness of granular layer $g = H_{gl}G_{gl}/2$, whereas Maheshwari and Khatri [8] considered the stiffness of granular layer $g = H_{gl}G_{gl}$.

Comparison with PLAXIS program

The present method is validated by comparing its results with the results from PLAXIS program. The strip footing is of length 20 m, width 1.0 m and flexural rigidity $EI = 281{,}300$ kN m^2 and subjected to uniformly distributed load of 100 kN/m^2. The thickness of the weak soil layer is 10 m. The thickness of the granular layer and its modulus of elasticity are 0.3 m and 20,000 kN/m^2, respectively. The end bearing granular piles diameters are 0.5 m and its spacing is 1.5 m. The modulus of elasticity of weak soil and granular piles are 6000 and 50,000 kN/m^2, respectively. Poisson's ratio of weak soil, granular layer and granular piles is taken equal to 0.25. Linear analysis is considered. Triangular elements of 15 nodes are used in the finite element analysis by PLAXIS program as shown in Fig. 5. The mesh has 364 elements and 3037 nodes. The linear elastic model under drained conditions, which is available in PLAXIS program library, used to model the weak soil, the stone column and the granular layer.

Figure 6 shows comparisons between the results of PLAXIS program and present program for untreated and treated cases. For untreated case, the results of the present method approximately equal to the results of PLAXIS program at the center of the beam while, at the edge of the beam the results of the present method smaller than that of PLAXIS program by 6.6% as shown in Fig. 6. For treated case, the results of the present program smaller than the results of PLAXIS program by approximately 11.1% and 10.1 at the center and the edge of the beam respectively.

Parametric study

The developed program used in a parametric study to show the effect of different parameters on the behavior of strip footing resting on granular layer over weak soil reinforced by granular piles in terms of vertical and differential displacements and bending moment. These parameters include number of granular piles, modular ratio (i.e., stiffness

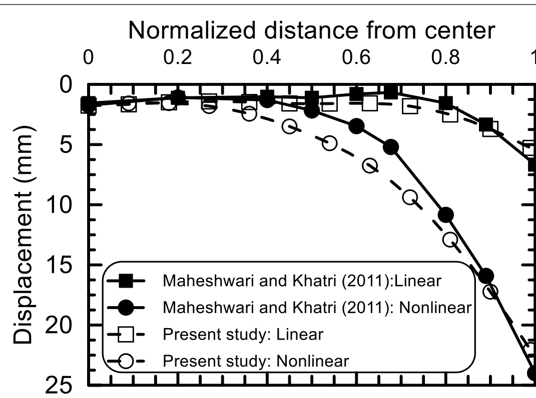

Fig. 4 Comparison between vertical displacements obtained by the present method and Maheshwari and Khatri [8] method

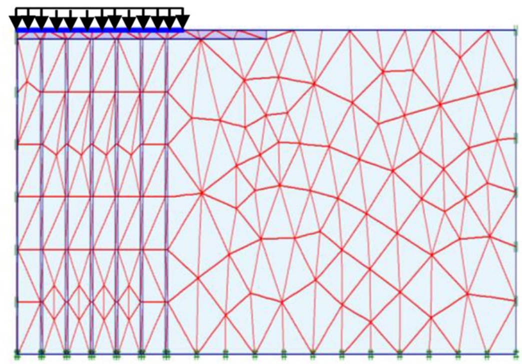

Fig. 5 Finite element mesh of PLAXIS program

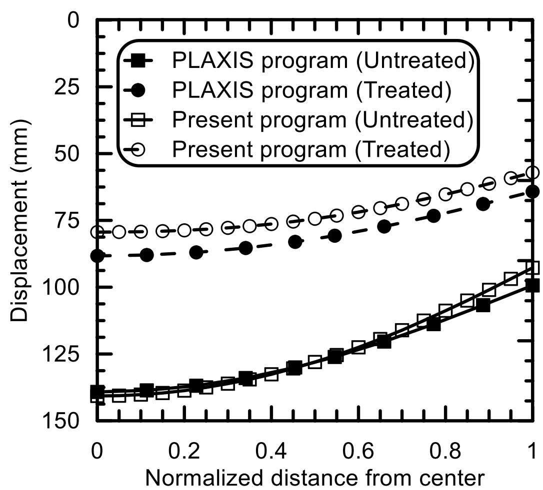

Fig. 6 Comparisons between the results of PLAXIS program and present program for untreated treated cases

of granular piles/stiffness of the weak soil), stiffness of granular layer, diameter of granular piles, length of granular piles, arrangement of granular piles, and granular piles of different diameters. The dimensions of the strip footing are 25 m length, 1.5 m width and 0.6 m thickness and its flexural rigidity is 567,000 kN m^2. The strip footing subjected to seven concentrated columns loads. The spacing between columns is 4 m and the loads of edge and interior columns are 150, 300 kN respectively. The nonlinear analysis is considered and the program as previously discussed calculates the ultimate bearing capacities of granular piles internally. The value of the influence depth is taken equal to 3 times the beam width. In all parametric study, only one parameter is changed, and all of the other parameters are held constant at the base values as presented in Table 1.

The results are presented in terms of dimensionless parameters as follows: normalized vertical displacement, $I_w = w/L$, normalized differential displacement, $I_{dw} = w_d/L$, and normalized bending moment, $I_m = mL/EI$ (where w is the vertical displacement, w_d is the differential displacement, m is the bending moment, L is the length of the strip footing, and EI is the flexural rigidity of the strip footing.

Table 1 Basic values of various parameters used in the parametric study

Parameter	Values
Thickness of weak soil	15 m
Elasticity modulus of weak soil	4 MPa
Poisson's ratio of weak soil	0.35
Submerged unit weight of weak soil	8 kN/m^3
Ultimate bearing capacity of weak soil	100 kPa
Number of granular piles	13
Diameter of granular piles	0.5 m
Length of granular piles	15 m
Poisson's ratio of granular piles	0.3
Elasticity modulus of granular piles	40 MPa
Angle of friction of granular piles	35^o
Thickness of granular layer	0.25 m
Elasticity modulus of granular layer	10 MPa
Poisson's ratio of granular layer	0.3
R_{fs}, R_{fgp}	1.0

Effect of granular piles number

Figures 7, 8 and 9 show the effect of the number of granular piles on the behavior of soil-strip footing system. The number of granular piles is varied from 0 to 25, whereas the other parameters are kept constant as presented in Table 1. Figure 7 shows comparison of the vertical displacements of strip footing without granular piles and with different numbers of granular piles. As expected, the vertical displacements of the strip footing reduce with the inclusion of granular piles in the weak soil. For example, the vertical and differential displacements decreases about 37.2 and 46.7% as the number of granular piles increases from 0 to 13 as shown in Figs. 7 and 8. The rate of decrease in differential displacement reduces as the number of granular piles increases as shown in Fig. 8.

The bending moment in the strip footing decreases as the number of granular piles increases as shown in Fig. 9. As the number of granular piles increases from 0 to 13, the value of the bending moment at the center of the strip footing is decreased by 50%. In

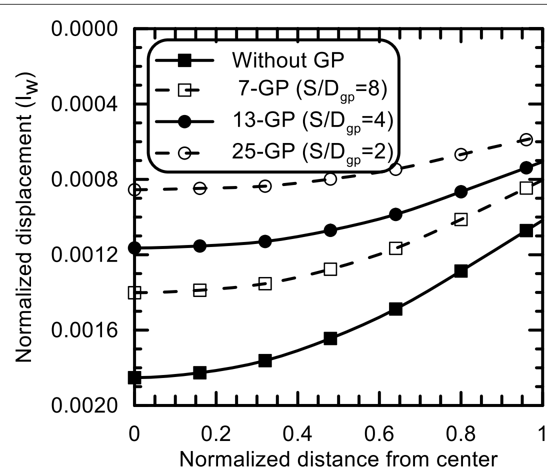

Fig. 7 Effect of granular piles number on the displacement of strip footing

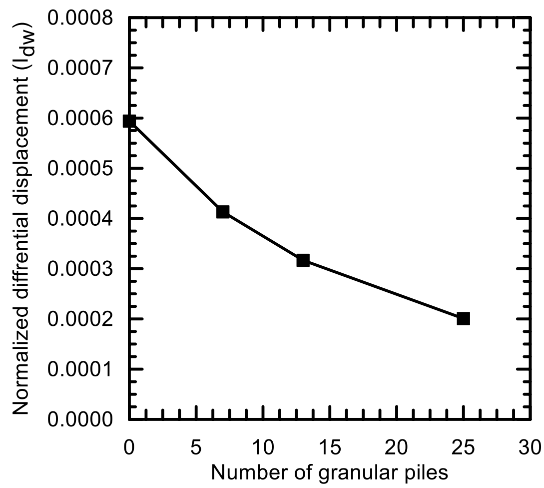

Fig. 8 Effect of granular piles number on the differential displacement of strip footing

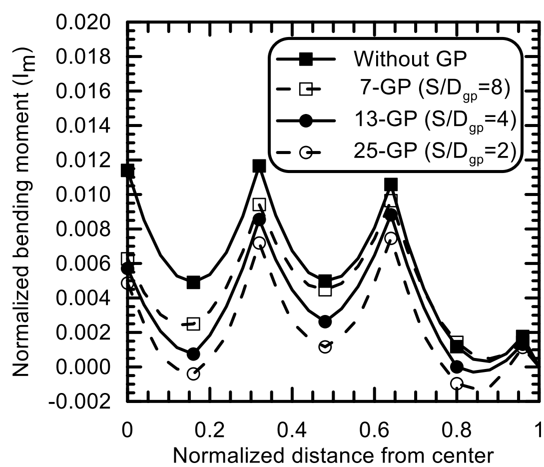

Fig. 9 Effect of granular pile number on the bending moment distribution of strip footing

addition, the decrease in the bending moment over the middle part of the strip footing is higher than that over the edge part.

Effect of modular ratio

In this section, the effect of soil stiffness and granular piles stiffness (i.e., the modular ratio, E_{gp}/E_s) on the behavior of soil-strip footing system is studied. Two cases are studied: case of varying the soil stiffness and remain the granular piles stiffness constant and case of varying the granular piles stiffness and remain the soil stiffness constant. For the two cases the modular ratio varied from 5 to 50. In the first case, soil stiffness are varied from $E_s = 0.8$ to 1, 2, 4 and 8 MPa while the granular piles stiffness, E_{gp}, remain constant as 40 MPa. In the second case, the granular piles stiffness are varied from $E_{gp} = 20$ to 40, 80, 160 and 200 MPa while the soil stiffness, E_s, remain constant as 4 MPa.

Figures 10 and 11 show the effect of soil stiffness on the vertical displacements and the bending moment of the strip footing. As shown in Figs. 10 and 11, the soil stiffness

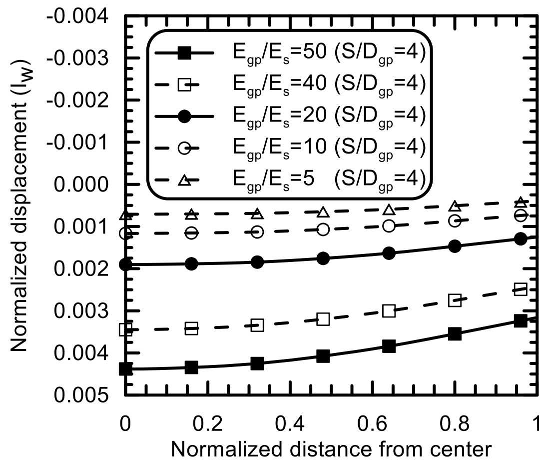

Fig. 10 Effect of soil stiffness on the vertical displacement of strip footing

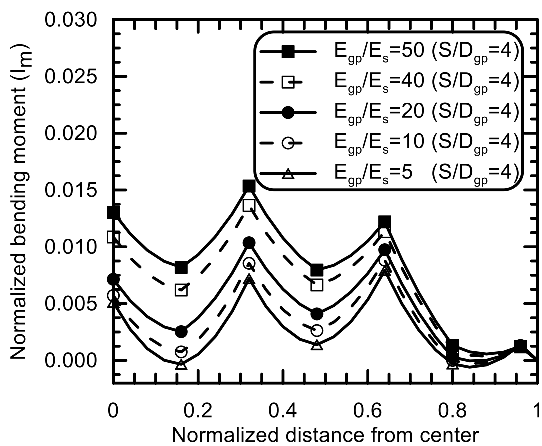

Fig. 11 Effect of soil stiffness on the bending moment distribution of strip footing

significantly influences the vertical and differential displacements and the distribution of the bending moments of the strip footing. The vertical and differential displacements are found to reduce with the increase in soil stiffness (i.e., decrease in the modular ratio) as shown in Fig. 10. A reduction in the maximum and differential displacements of the strip footing of about 83.8 and 73.6%, respectively, is found as the soil stiffness increases from 0.8 to 8 MPa (i.e., the modular ratio decreases from 50 to 5).

The bending moment in the strip footing decreases as the soil stiffness increases as shown in Fig. 11. As the soil stiffness increases from 0.8 to 8 MPa (i.e., the modular ratio decreases from 50 to 5), the value of the bending moment at the center of the strip footing decreases by 60.8%. Also, the decrease in the bending moment over the edge part of the strip footing is smaller than the decrease in the bending moment over the middle part.

The vertical and differential displacements decrease as the stiffness of granular piles increases as shown in Fig. 12. A reduction in the maximum and differential displacements of the strip footing by 52.2 and 55.2% respectively as the granular piles stiffness increases from 20 to 200 MPa (i.e., the modular ratio increases from 5 to 50).

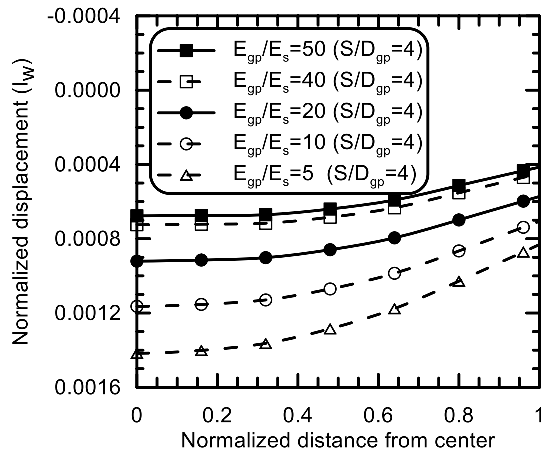

Fig. 12 Effect of granular pile stiffness on the vertical displacement of strip footing

As shown in Fig. 13, the bending moment in the strip footing decreases as the stiffness of the granular piles increases. As the granular piles stiffness increases from 20 to 200 MPa (i.e., the modular ratio increases from 5 to 50, the value of the bending moment at the center of the strip footing decreases by 67.2%. In addition, the decrease in the bending moment over the edge part is smaller than the decrease in the bending moment over the rest of the strip footing.

Comparison between the effect of soil stiffness and granular piles stiffness on the differential displacement and the bending moment at the center of the strip footing are shown in Figs. 14 and 15 respectively. Referring to these figures, it is observed that: (1) compared with the effect of granular piles stiffness, the effect of soil stiffness on differential displacement and flexural performance of strip footing is more significant, (2) the effect of granular piles stiffness on the behavior of the strip footing decreases as the modular ratio increases. This effect is approximately negligible especially when the modular ratio is greater than 40 and (3) with respect to soil stiffness, the differential displacement and the bending moment at the center of the strip footing increase as the modular ratio increases (i.e., soil stiffness decreases). Inversely, with respect to granular piles

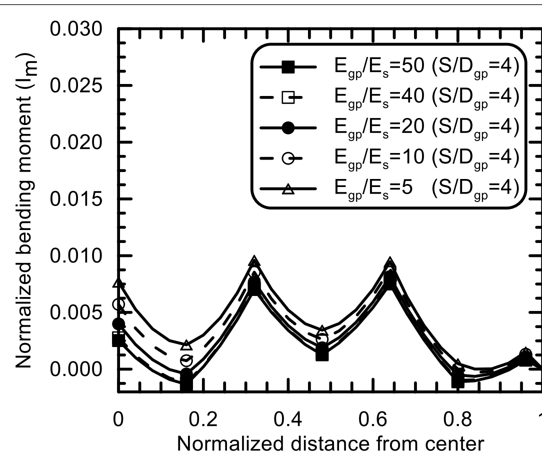

Fig. 13 Effect of granular pile stiffness on the bending moment distribution of strip footing

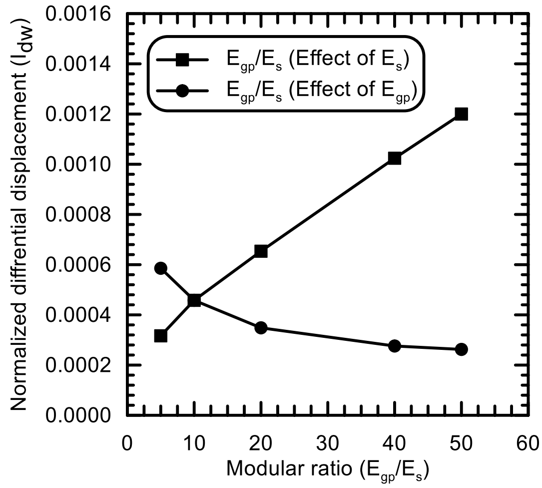

Fig. 14 Effect of soil stiffness and granular pile stiffness on the differential displacement of strip footing

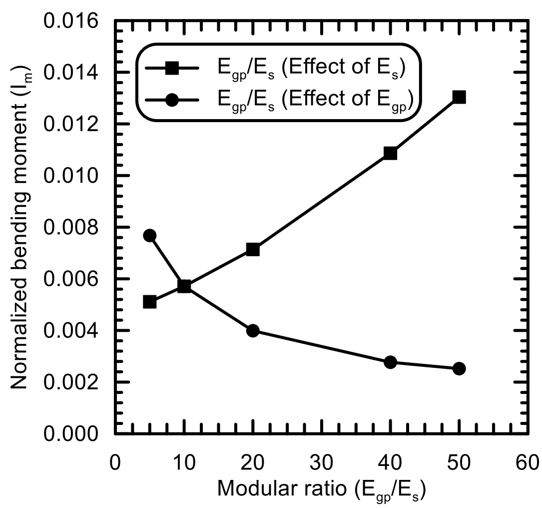

Fig. 15 Comparison between the effect of soil stiffness and granular pile stiffness on the bending moment at center of strip footing

stiffness the differential displacement and the bending moment at the center of the strip footing decrease as the modular ratio increases (i.e., granular piles stiffness increases). Therefore and to prevent the confusion, it is better to study the effect of soil stiffness and granular piles stiffness on the behavior of soil-strip footing system separately instead of studying the effect of modular ratio.

Effect of granular layer stiffness

In the present study, Eq. (11) used to calculate the stiffness of the granular layer. The stiffness, *g*, is proportional to the modulus of elasticity and thickness of the granular layer as shown in Eq. (11). Thus, as the modulus of elasticity or the thickness of the granular layer increases, the value of *g* also increases. The effect of the modulus of elasticity of the granular layer, represented as a ratio from the soil modulus of elasticity, E_{gl}/E_s, on the behavior of soil-strip footing system is studied by varying the E_{gl}/E_s ratio from 2.5 to 50 as shown in Figs. 16, 17 and 18. As the modulus ratio, E_{gl}/E_s, increases (i.e., the stiffness, *g*, increases), the vertical displacement at the center of the strip footing decreases, while it increases at the edge and therefore, the differential displacement decreases as shown in Figs. 16 and 17 respectively. As the modulus ratio, E_{gl}/E_s, increases from 2.5 to 50, a reduction in the vertical displacement at the center of the strip footing by 3.2% and an increase in the edge vertical displacement by 7.8% (Fig. 16) and this leads to a reduction in differential displacement by about 19.2% (Fig. 17). This means that higher values of modulus of elasticity of granular layer are preferable to avoid differential displacement.

The increase of the modulus ratio, E_{gl}/E_s, result in a decrease in the bending moment in the strip footing especially at the points between columns while, the changes are smaller at the columns as shown in Fig. 18. As the modulus ratio, E_{gl}/E_s, increases from 2.5 to 50, the bending moment at 6.5 m distance from the edge decreases by about 29.7% while it decreases by about 14.3% at 4.5 m distance from the edge of the strip footing.

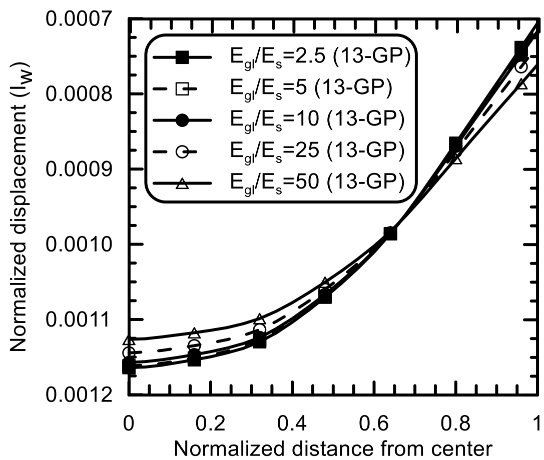

Fig. 16 Effect of granular layer thickness on the displacement of strip footing

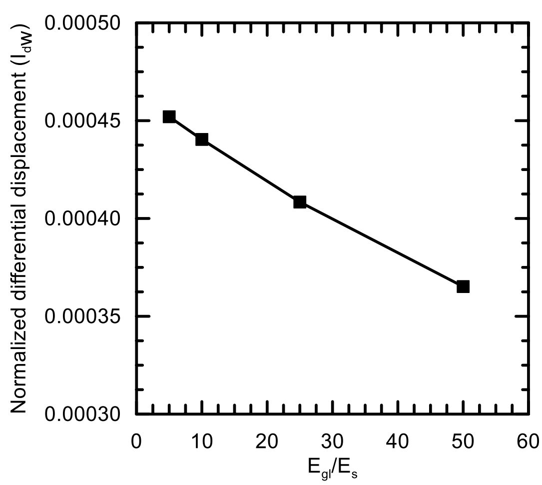

Fig. 17 Effect of granular layer thickness on the differential displacement of strip footing

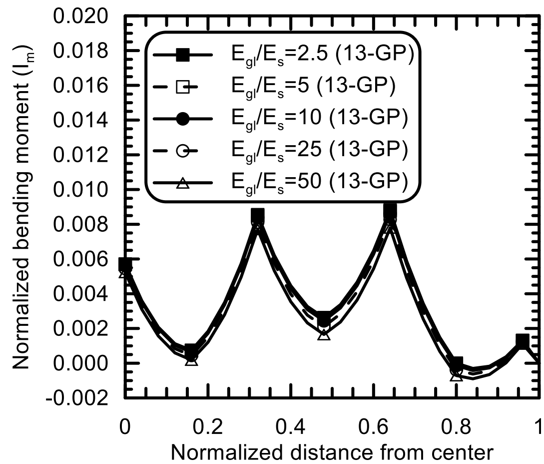

Fig. 18 Effect of granular layer thickness on the bending moment of strip footing

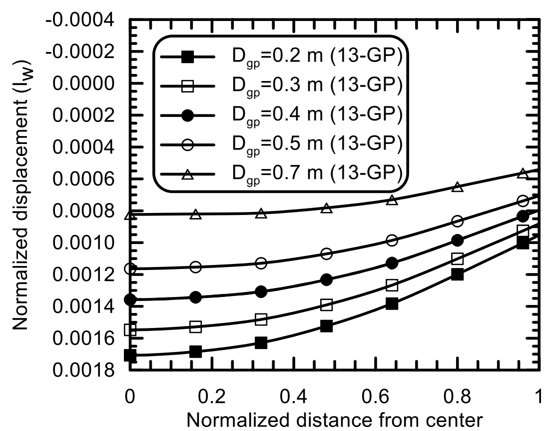

Fig. 19 Effect of granular pile diameter on the vertical displacement of strip footing

Effect of granular pile diameter

Figures 19, 20 and 21 show the effect of the granular piles diameter on the behavior of the soil-strip footing system. The diameter of the granular piles varied from 0.2 to 0.7 m and the other parameters are kept constant at its basic values as shown in Table 1. It is important to note that at the same number of granular piles, the changes in the granular piles diameters leads to decrease in the spacing to diameter ratio (i.e., S/D_{gp}). Referring to Figs. 19 and 20, as the granular piles diameter increases, the vertical and differential displacements decrease. The increase of granular piles diameter from 0.2 to 0.5 m result in a decrease by about 31.8% in the maximum displacement at the center of the strip footing and a decrease by about 39.2% in the differential displacement.

Figure 21 shows the influence of diameter of granular piles on the bending moment distribution in the strip footing. The bending moment in the strip footing decreases as the granular piles diameter increases especially at the middle part of the strip footing. An increase of the granular piles diameters from 0.2 to 0.5 m causes a decrease in the bending moment at the center of the footing by about 44%.

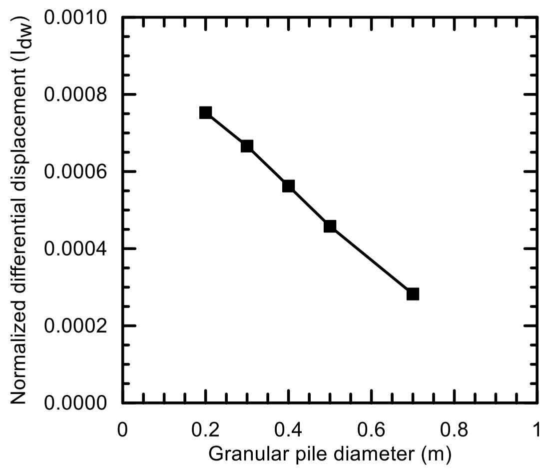

Fig. 20 Effect of granular pile diameter on the differential displacement of strip footing

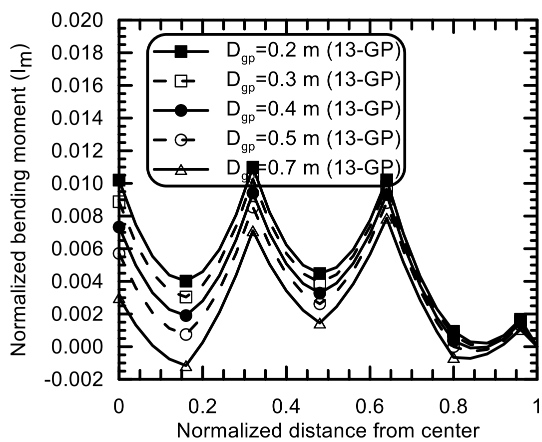

Fig. 21 Effect of granular pile diameter the bending moment of strip footing

Effect of granular pile length

Granular piles may be fully penetrated and resting on strong soil layer (i.e., end bearing granular piles) or partially penetrated (i.e., floating granular piles). The effect of the ratio L_{gp}/H (where L_{gp} is the length of granular pile and H is the thickness of weak soil layer to the rigid base) on the behavior of the soil-strip footing system is shown in Figs. 22, 23 and 24. Typical values of L_{gp}/H ratio used in the study are 0.2, 0.4, 0.6, 0.8 and 1.0, whereas the other parameters remain constant at its basic values. As shown in Figs. 22 and 23, the effect of L_{gp}/H ratio on the vertical and differential displacements of the strip footing is minimal for L_{gp}/H ratio greater than 0.4. A decrease in the L_{gp}/H ratio from 1.0 to 0.4 causes an increase in the maximum and differential displacements by 7.2 and 8.8%, respectively and subsequently an increase by about 7.6 and 9.9% in the maximum and differential displacements respectively as the L_{gp}/H ratio decreases from 0.4 to 0.2. Kolekar and Murty [16] reported same observations from numerical analysis for single floating granular pile.

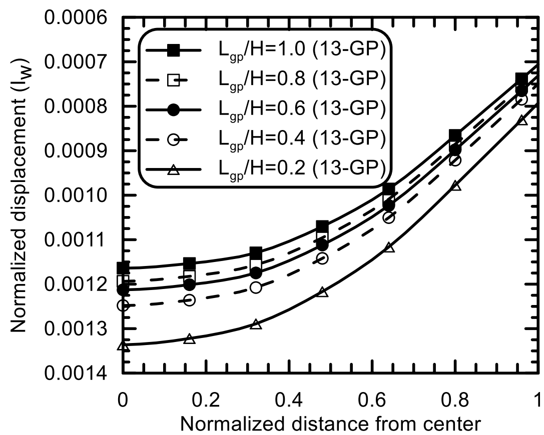

Fig. 22 Effect of granular pile length on the displacement of strip footing

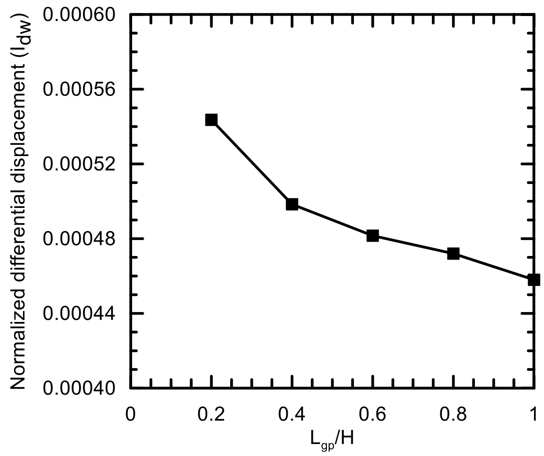

Fig. 23 Effect of granular pile length on the displacement of strip footing

Figure 24 shows the effect of L_{gp}/H ratio on the distribution of the bending moment in the strip footing. The effect of L_{gp}/H ratio on the bending moment distribution is minimal especially over the edge parts of the strip footing. A reduction of L_{gp}/H ratio from 1.0 to 0.4 results in an increase in the value of the bending moment at the center of the strip footing by 11.2%, whereas an increase of 12% in the bending moment at the center of strip footing as the L_{gp}/H ratio decreases from 0.4 to 0.2.

Effect of granular pile arrangement

The aim of this section is to determine the optimal method of arranging the granular piles beneath the strip footing that produce the minimum vertical and differential displacements as well as the induced bending moment in the strip footing. Five different arrangements for granular piles are investigated as shown in Fig. 25. Typical values of input parameters are as presented in Table 1.

Figures 26, 27 and 28 show the effect of different arrangements of granular piles (i.e., GPA1, GPA2, GPA3, GPA4 and GPA5) on the behavior of soil-strip footing system. The

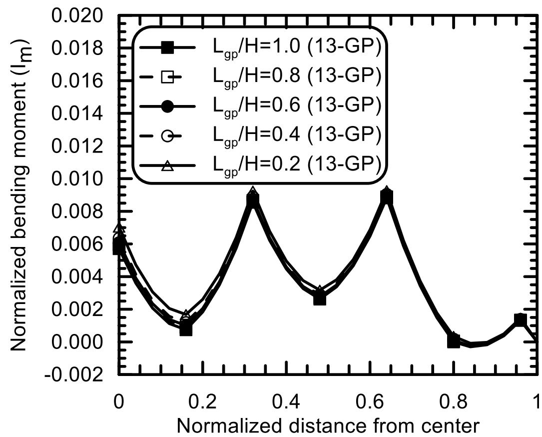

Fig. 24 Effect of granular pile length on the bending moment in strip footing

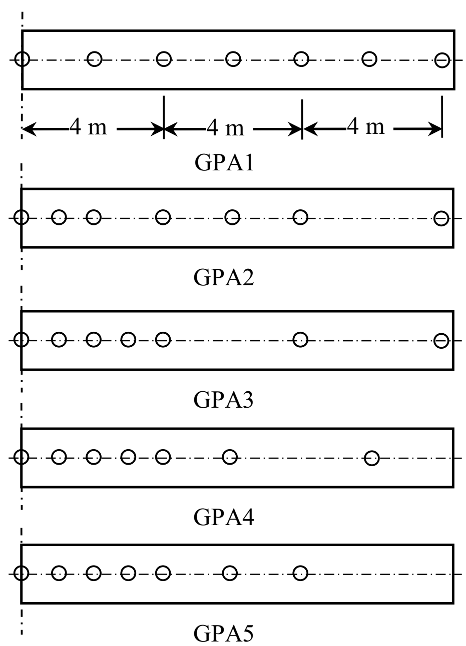

Fig. 25 Studied cases of granular piles arrangements (No. of granular piles = 13)

differential displacement is the difference between the points of maximum and minimum vertical displacements along the strip footing length. The effect of the granular pile arrangements on the differential displacement is more significant that its effect on the vertical displacement. As shown in Figs. 26 and 27, the granular pile arrangement, GPA5, has the smallest vertical and differential displacements. Comparing to the uniformly granular pile arrangement, GPA1, the granular piles arrangement, GPA5, causes a decrease in the maximum and differential displacements by 12.7 and 80%, respectively.

Figure 28 shows the effects of different granular piles arrangement on the distribution of bending moment induced in the strip footing. It is clear from Fig. 28 that higher effect on the bending moment in the middle third of the strip footing and lower effect on

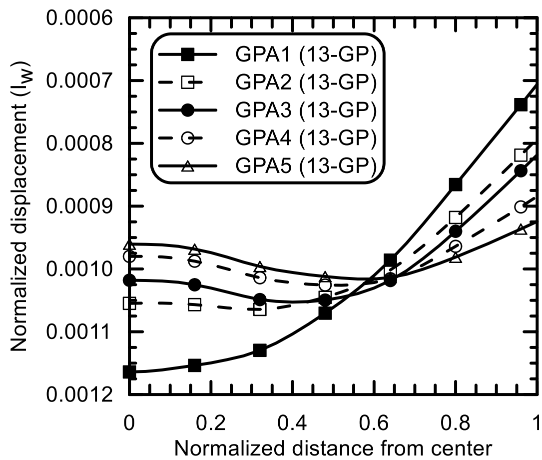

Fig. 26 Effect of granular piles arrangements on the displacement of strip footing

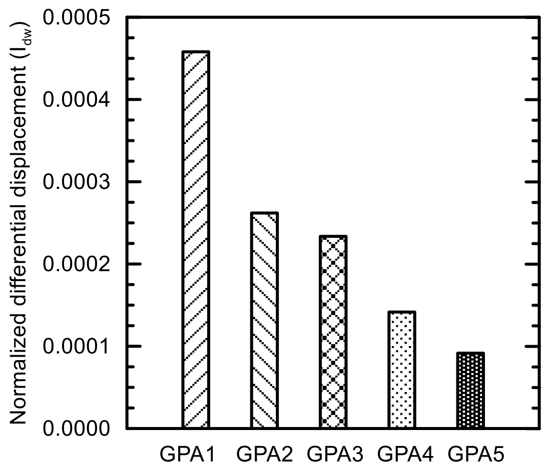

Fig. 27 Effect of granular piles arrangements on the differential displacement of strip footing

the bending moments in edge thirds of the strip footing due to different granular piles arrangements. The granular pile arrangement, GPA5, has the smallest value of positive bending moment and the highest value of negative bending moment as shown in Fig. 28. Comparing to the uniformly granular piles arrangement, GPA1, the arrangement, GPA5, causes a decrease in the bending moments at the center and at 4 m distance from the center by 97 and 41.5%, respectively.

Conclusions

This paper presents a method for analysis of strip footing resting on granular layer over weak soil reinforced by end bearing or floating granular piles. The method of analysis taking into account the shear effect or the continuity of the granular piles-weak soil composite and the nonlinear behavior of weak soil and granular piles. Comparisons between the results of the present analysis with the field measurements and the results of other existing analysis method show good agreement. Based on the observed results, the following conclusions are drawn:

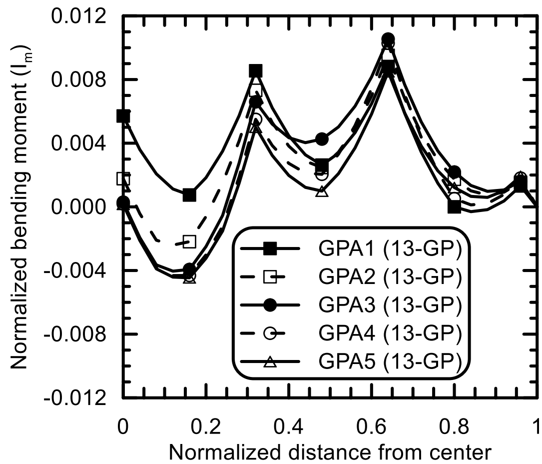

Fig. 28 Effect of granular pile arrangements on the bending moment of strip footing

1. The use of even small number of granular piles with the weak soil enhances the behavior of the strip footing. The vertical and differential displacements and the induced bending moments in the strip footing reduces significantly as the number of granular piles increases.
2. The increase in soil stiffness and granular piles stiffness enhances the performance of strip footing. Compared with the effect of granular pile stiffness, the effect of soil stiffness on the vertical and differential displacements and bending moment of the strip footing is more significant.
3. The increase in the granular layer stiffness enhances the behavior of strip footing. The effect of granular layer stiffness on differential displacement is more significant than its effect on the vertical displacement and the bending moment. Therefore, higher values of granular layer stiffness are preferable to avoid differential displacement.
4. For the same number of granular piles, the vertical and differential displacements and the bending moment of the strip footing reduces significantly as the diameters of granular piles increases.
5. Compared to the end bearing granular piles, the use of floating granular piles up to a length of $0.4H$ cause an insignificant increase in the vertical and differential displacements and the bending moment in the strip footing.
6. Compared to uniform arrangement of granular piles, the concentrated arrangement of granular piles underneath the heavily loaded part of the strip footing is more effective in reducing the vertical and differential displacements and the induced bending moment in the strip footing.

Authors' contributions

BE carried out the research work, literature review, method of analysis, finite element formulation, results and discussion, parametric study and wrote the manuscript. ME was shared his knowledge at every stages of this paper especially in preparing the manuscript and at the conclusions part. Both authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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